Q.P. Code : 60865

Second Semester M.Sc. Degree Examination, July 2019

(CBCS Scheme)

Mathematics

Paper M 205 T – NUMERICAL ANALYSIS – I

Time : 3 Hours] *[Max. Marks : 70*

Instructions :

1) *Answer any* ***FIVE*** *full questions.*
2) *All questions carry equal marks.*

1. (a) State Descarte's rule of signs and Sturm's theorem using strum sequence. Find the number of real and complex roots of $4x^4 + 2x^2 - 1 = 0$.

   (b) Prove or disprove that the Newton-Raphson method for finding a simple root of $f(x) = 0$ has a quadratic convergence whereas linearly for finding a multiple root. (7 + 7)

2. (a) Extract a quadratic factor of the form $px^2 + qx + 1 = 0$ from $x^3 - x - 1 = 0$ $(p_0 = 0.5 = q_0)$.

   (b) Find roots of $2x - \cos x - 3 = 0$ with $x_0 = \pi/2$ correct to four decimal places using Aitkene's Δ^2 - method. (7 + 7)

3. (a) Define an $n \times n$ order tri-diagonal and develop a recursion algorithm to this system. Also solve by using algorithm $[A:B] = \begin{bmatrix} 2 & 1 & 0 & 0:1 \\ 2 & 3 & 1 & 0:2 \\ 0 & 1 & 4 & 2:3 \\ 0 & 0 & 1 & 3:4 \end{bmatrix}$

   (b) Explain ill-conditioning with examples. Show that the Hilbert matrix of order 3 is highly ill-conditioned by finding its condition number. (7 + 7)

4. (a) For the solution of the system of equation

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$$

   set up SOR method. Find the optimal relaxation factor.

   (b) Solve the equation $y\cos(xy) + 1 = 0$ $\sin(xy) + x - y = 0$ assuming the initial approximation (1, 2) by homotopy continuation method. (7 + 7)

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5. (a) Derive the Hermite interpolation formula for the data values $(x_i, f(x_i), f'(x_i))$, $i = 0, 1, 2, \ldots\ldots, n$.

(b) Find the rational approximation $R_{2,3}$ for the function $f(x) = \sin x$. (7 + 7)

6. (a) Derive Newton's bivariate interpolation polynomial for equispaced points, hence find $f(0.25, 0.75)$ using linear interpolation for the data :

y \ x	0	1
0	1	1.4142
1	1.7820	1

(b) Obtain the cubic spline approximation for the function defined by the data :

x :	0	1	2	3
f (x) :	1	2	33	244

with $M(0) = 0$, $M(3) = 0$. Find an estimate of $f(2.5)$. (7 + 7)

7. (a) Evaluate $\int_{-1}^{1} e^{-x^2} \cos x\, dx$ using the Gauss-Legendre 2 and 3 point quadratur formula. Compare the results with exact solutions.

(b) Derive Gauss-Hermitz 2 and 3 point quadrature formula and hence evaluate

$$\int_{-\infty}^{\infty} \frac{e^{-x^2}}{1+x^2} dx.$$

(7 + 7)

8. (a) Derive Simpson's rule for $\int_a^b \int_c^d f(x, y)\, dx\, dy$ for two subinterval.

(b) Evaluate $\int_0^1 \int_0^2 \frac{2xy}{(1+x^2)(1+y^2)} dy\, dx$ using trapezoidal rule with $h = k = 0.5$ and compare results with exact solutions. (7 + 7)